

# Anomalous $U(1)$ , holomorphy, supersymmetry breaking and dilaton stabilization

Nikolaos Irges <sup>1</sup>

*Institute for Fundamental Theory,*

*Department of Physics, University of Florida*

*Gainesville FL 32611, USA*

## Abstract

We argue that in certain models with family symmetries the implementation of the alignment mechanism for the suppression of the flavor changing neutral currents requires mass matrices with holomorphic zeros in the down quark sector. Holomorphic zeros typically open flat directions that potentially spoil the uniqueness of the supersymmetric vacuum. We then present an anomalous  $U(1)$  model without holomorphic zeros in the quark sector that can reproduce the fermion mass hierarchies, provided that  $\tan \beta$  is of order one. To avoid undesired flavor changing neutral currents we propose a supersymmetry breaking mechanism and a dilaton stabilization scenario that result in degenerate squarks at  $M \sim M_{GUT}$  and a calculable low energy spectrum. We present the numerical predictions of this model for the Higgs mass for different values of  $M$  and  $\tan \beta$ .

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<sup>1</sup>e-mail address: irges@phys.ufl.edu

# 1 Introduction

Recently[1], it was shown that in models of fermion masses with family symmetries, the assumption of no holomorphic zeros in the quark sector of the superpotential uniquely determines the form of the  $U(1)$  that reproduces the data, to  $Y_X \equiv X + Y_F \equiv X + Y^{(1)} + Y^{(2)}$ , with

$$Y^{(1)} = \frac{1}{5}(2Y + V)(2, -1, -1) \quad \text{and} \quad Y^{(2)} = -\frac{1}{2}(V + 3V')(1, 0, -1), \quad (1.1)$$

where the vectors  $(2, -1, -1)$ ,  $(1, 0, -1)$  show the family dependence,  $Y$  is hypercharge and  $V, V'$  are the other two anomaly free  $U(1)$ 's in the **27** of  $E_6$ .  $X$  is a family independent, anomalous trace which had to be added to implement the correct intrafamily hierarchy [1],[2]. The matter content is 3 families of fields filling out the **27** of  $E_6$ , except the singlet outside  $SO(10)$ . One interesting prediction of this model is the neutrino mixing matrix that can be conveniently expressed in terms of the Cabbibo angle  $\lambda_c$ , as:

$$\begin{pmatrix} 1 & \lambda_c^3 & \lambda_c^3 \\ \lambda_c^3 & 1 & 1 \\ \lambda_c^3 & 1 & 1 \end{pmatrix}, \quad (1.2)$$

consistent with the atmospheric neutrino data coming from SuperK and the non-adiabatic MSW solution to the solar neutrino flux anomaly. Any complete supersymmetric model of masses however, has to also give an explanation for the suppression of flavor changing neutral current (fcnc) processes, which generically get large contribution in the presence of an anomalous  $U(1)$ [3], due to the nonzero  $D$ -term associated with it.

A possibility of solving this problem is when there is alignment between the up and down sectors [4]. One striking property of this mechanism is that there is no need for degenerate squarks after supersymmetry breaking, to suppress fcnc. We will show soon that alignment can take place only if the mass matrices in the up and down quark sectors have the form:

$$Y_u^{\text{align}} = \begin{pmatrix} \lambda_c^8 & \lambda_c^5 & \lambda_c^3 \\ * & \lambda_c^4 & \lambda_c^2 \\ * & * & 1 \end{pmatrix} \quad \text{and} \quad Y_d^{\text{align}} = \begin{pmatrix} \lambda_c^4 & 0 & \lambda_c^3 \\ 0 & \lambda_c^2 & \lambda_c^2 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1.3)$$

where  $*$  means an entry which is a holomorphic zero or not. Even though this is a possible

solution to the fcnc problem, we choose not to favor it because holomorphic zeros tend to destabilize the vacuum, as was argued in [5].

In the case of absence of holomorphic zeros from the quark sector of the superpotential, the contributions of the up and down sectors to the CKM matrix is the same, alignment does not take place and therefore to suppress fcnc, a supersymmetry mechanism which generates degenerate squarks is necessary. This leads us to the scenario of dilaton dominated supersymmetry breaking, which is a mechanism that can give degenerate squark masses at the scale  $M \sim M_{GUT}$ . The main difficulties of constructing a predictive model of this kind is to solve the problem of dilaton stabilization and to give an explanation for why the  $D$ -term contributions to the soft masses is suppressed.

In section II, we show that the most general form of matrices that can implement the alignment mechanism is that of 1.3, which contains undesired, in our point of view, holomorphic zeros. In section III, we turn into models with minimum number of holomorphic zeros and we show that it is possible to stabilize the dilaton via non perturbative corrections to the Kähler potential and at the same time suppress the  $D$ -term contributions to the soft masses, so that the (family universal) contribution of the dilaton  $F$ -term dominates the squark masses. We give an explicit example of a globally supersymmetric model that has these properties and finally we argue that supergravity corrections do not modify this picture.

## 2 Holomorphic zeros and alignment

We consider models with  $N + 1$  family  $U(1)$  symmetries. If  $N = 0$ , there is only one such  $U(1)$ , which has to be anomalous, as mentioned in the introduction. Even though we do not have a proof for it, cancelation of anomalies containing hypercharge, strongly indicate that it is the trace of the  $U(1)$  that carries all of the anomaly, leaving its traceless part anomaly free. If  $N \geq 1$ , for the same reason, without loss of generality, we will assume that all of the anomaly is contained in a family independent  $U(1)$ , leaving the other  $N$  traceless and anomaly free. In both cases we will call the anomalous part  $X$ . Also, in a model with  $N + 1$   $U(1)$ 's, we assume

$N + 1$  order parameter fields  $\theta_\alpha$ , ( $\alpha = 0, \dots, N$ ), that take vevs and break those  $U(1)$ 's. A gauge invariant term in the superpotential has the form:

$$\left(\frac{\theta_0}{M}\right)^{n_0^{i_1 i_2 i_3 \dots i_I}} \dots \left(\frac{\theta_N}{M}\right)^{n_N^{i_1 i_2 i_3 \dots i_I}} \mathbf{I}_{i_1 i_2 i_3 \dots i_I}. \quad (2.4)$$

We have displayed in the invariant  $\mathbf{I}$  and the exponents the family indices explicitly. The invariance of this term under the  $U(1)$ 's allows us to compute the powers  $n_\alpha^{i_1 \dots i_I}$  from the matrix equation  $\mathbf{n} = -\mathcal{A}^{-1} \mathbf{Y}^{(I)}$ , or explicitly:

$$\begin{pmatrix} n_0^{i_1 i_2 i_3 \dots i_I} \\ . \\ . \\ . \\ n_N^{i_1 i_2 i_3 \dots i_I} \end{pmatrix} = -\mathcal{A}^{-1} \begin{pmatrix} X(\mathbf{I}_{i_1 i_2 i_3 \dots i_I}) \\ Y^{(1)}(\mathbf{I}_{i_1 i_2 i_3 \dots i_I}) \\ . \\ . \\ . \\ Y^{(N)}(\mathbf{I}_{i_1 i_2 i_3 \dots i_I}) \end{pmatrix}, \quad (2.5)$$

where we have introduced the following notation:  $\mathbf{n}$  is an  $(N + 1) \times 1$  column vector with the powers of the  $\theta$  fields,  $\mathbf{Y}^{(I)}$  is an  $(N + 1) \times 1$  column vector with the charges of the standard model invariant  $\mathbf{I}_{i_1 i_2 i_3 \dots i_I}$  under the  $N + 1$   $U(1)$ 's,  $\mathcal{A}$  is the  $(N + 1) \times (N + 1)$  matrix whose first row is the  $X$  charges of  $\theta_\alpha$  and its  $a$ 'th row ( $a = 1, \dots, N$ ) is the  $Y^{(a)}$  charges of  $\theta_\alpha$ .  $\mathcal{A}^{-1}$  is assumed to be of such form so that all the elements of its first column are 1. This means that all the  $\theta$  fields ( $N + 1$  of them) take vacuum expectation values at the same scale.<sup>2</sup> We denote the common vev  $\langle \theta_\alpha \rangle / M$  by  $\lambda_c$  and later identify it with the Cabbibo angle. We set the notation for the Abelian charges of the observed quarks under the  $a$ 'th non-anomalous  $U(1)$ :

	1st family	2nd family	3rd family
$\mathbf{Q}$	$q_1^{[a]}$	$q_2^{[a]}$	$-q_1^{[a]} - q_2^{[a]}$
$\bar{\mathbf{u}}$	$u_1^{[a]}$	$u_2^{[a]}$	$-u_1^{[a]} - u_2^{[a]}$
$\bar{\mathbf{d}}$	$d_1^{[a]}$	$d_2^{[a]}$	$-d_1^{[a]} - d_2^{[a]}$

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<sup>2</sup>We will keep this assumption until the end of this paper, because it makes the discussion on mass matrices easier. We could relax this assumption and still arrive at the same conclusions.

In the case of a single, anomalous  $U(1)$ , the above table gives the charges of its traceless part.

It is useful to introduce the quantities

$$Q_{12}^{[a]} \equiv 2q_1^{[a]} + q_2^{[a]} \quad \text{and} \quad Q_{21}^{[a]} \equiv 2q_2^{[a]} + q_1^{[a]}, \quad (2.6)$$

$$U_{12}^{[a]} \equiv 2u_1^{[a]} + u_2^{[a]} \quad \text{and} \quad U_{21}^{[a]} \equiv 2u_2^{[a]} + u_1^{[a]}, \quad (2.7)$$

$$D_{12}^{[a]} \equiv 2d_1^{[a]} + d_2^{[a]} \quad \text{and} \quad D_{21}^{[a]} \equiv 2d_2^{[a]} + d_1^{[a]}. \quad (2.8)$$

In the up and down quark sectors we get the Yukawa matrices

$$Y_u = \lambda_c^{N^{[u]}} \begin{pmatrix} \lambda_c^{M+K} & \lambda_c^{P+K} & \lambda_c^{0+K} \\ \lambda_c^{M+L} & \lambda_c^{P+L} & \lambda_c^{0+L} \\ \lambda_c^{M+0} & \lambda_c^{P+0} & \lambda_c^{0+0} \end{pmatrix} \quad \text{and} \quad Y_d = \lambda_c^{N^{[d]}} \begin{pmatrix} \lambda_c^{R+K} & \lambda_c^{T+K} & \lambda_c^{0+K} \\ \lambda_c^{R+L} & \lambda_c^{T+L} & \lambda_c^{0+L} \\ \lambda_c^{R+0} & \lambda_c^{T+0} & \lambda_c^{0+0} \end{pmatrix}. \quad (2.9)$$

$N^{[u]}$ ,  $N^{[d]}$  are defined as the total powers appearing at the 33 position of the corresponding

mass matrix which we always pull out front. Also,

$$\begin{pmatrix} K \\ L \\ M \\ P \\ R \\ T \end{pmatrix} = \sum_{\alpha} \begin{pmatrix} K_{\alpha} \\ L_{\alpha} \\ M_{\alpha} \\ P_{\alpha} \\ R_{\alpha} \\ T_{\alpha} \end{pmatrix}, \quad \text{where} \quad \begin{pmatrix} K_{\alpha} \\ L_{\alpha} \\ M_{\alpha} \\ P_{\alpha} \\ R_{\alpha} \\ T_{\alpha} \end{pmatrix} = -(\mathcal{A}^{-1})_{\alpha\beta} \begin{pmatrix} Q_{12}^{[\beta]} \\ Q_{21}^{[\beta]} \\ U_{12}^{[\beta]} \\ U_{21}^{[\beta]} \\ D_{12}^{[\beta]} \\ D_{21}^{[\beta]} \end{pmatrix}. \quad (2.10)$$

As usual,  $\alpha, \beta = 0, \dots, N$  and summation over  $\beta$  is implied.

The quark matrices are diagonalized as

$$Y_u = V_L^{u\dagger} M_u V_R^u, \quad Y_d = V_L^{d\dagger} M_d V_R^d. \quad (2.11)$$

From the above and 2.9, which imply  $K = 3$ ,  $L = 2$  with  $K_{\alpha}, L_{\alpha} \geq 0$ , we see that our notation for  $\lambda_c$  is fully justified. Similarly, for the squarks we have mass matrices associated with the soft terms  $\tilde{m}_{ij}^2 \tilde{\mathbf{q}}_i^* \tilde{\mathbf{q}}_j$ , which can be diagonalized as

$$\tilde{Y}_{LL}^{u,d} = \tilde{V}_L^{u,d\dagger} \tilde{M}_{LL}^{u,d} \tilde{V}_L^{u,d}, \quad \tilde{Y}_{RR}^u = \tilde{V}_R^{u\dagger} \tilde{M}_{RR}^u \tilde{V}_R^u, \quad \tilde{Y}_{LL}^d = \tilde{V}_L^{d\dagger} \tilde{M}_{RR}^d \tilde{V}_L^d. \quad (2.12)$$

The most stringent experimental limits on the entries of these matrices (coming primarily from the neutral meson mixing data) are [4]:

$$(K_L^d)_{12} = (V_L^d \tilde{V}_L^{d\dagger})_{12} = \lambda_c^{m_{12}^d} \leq \lambda_c^3, \quad \text{where}$$

$$(K_L^d)_{12} = \max \left[ (\tilde{V}_L^{d\dagger})_{12}, (V_L^{d\dagger})_{12}, (V_L^{d\dagger})_{13} \cdot (\tilde{V}_L^{d\dagger})_{32} \right], \quad (2.13)$$

$$(K_R^d)_{12} = (V_R^d \tilde{V}_R^{d\dagger})_{12} = \lambda_c^{m_{12}^{R_d}} \leq \lambda_c^3, \quad \text{where}$$

$$(K_R^d)_{12} = \max \left[ (\tilde{V}_R^{d\dagger})_{12}, (V_R^{d\dagger})_{12}, (V_R^{d\dagger})_{13} \cdot (\tilde{V}_R^{d\dagger})_{32} \right], \quad (2.14)$$

If there are no supersymmetric zeros in the mass matrices, we can explicitly diagonalize them.

We obtain

$$V_L^u = \begin{pmatrix} 1 & \lambda_c^{<K_\alpha - L_\alpha>} & \lambda_c^{<K_\alpha>} \\ \lambda_c^{<K_\alpha - L_\alpha>} & 1 & \lambda_c^{<L_\alpha>} \\ \lambda_c^{<K_\alpha>} & \lambda_c^{<L_\alpha>} & 1 \end{pmatrix}, \quad V_R^u = \begin{pmatrix} 1 & \lambda_c^{<M_\alpha - P_\alpha>} & \lambda_c^{<M_\alpha>} \\ \lambda_c^{<M_\alpha - P_\alpha>} & 1 & \lambda_c^{<P_\alpha>} \\ \lambda_c^{<M_\alpha>} & \lambda_c^{<P_\alpha>} & 1 \end{pmatrix} \quad (2.15)$$

$$V_L^d = \begin{pmatrix} 1 & \lambda_c^{<K_\alpha - L_\alpha>} & \lambda_c^{<K_\alpha>} \\ \lambda_c^{<K_\alpha - L_\alpha>} & 1 & \lambda_c^{<L_\alpha>} \\ \lambda_c^{<K_\alpha>} & \lambda_c^{<L_\alpha>} & 1 \end{pmatrix}, \quad V_R^d = \begin{pmatrix} 1 & \lambda_c^{<R_\alpha - T_\alpha>} & \lambda_c^{<R_\alpha>} \\ \lambda_c^{<R_\alpha - T_\alpha>} & 1 & \lambda_c^{<T_\alpha>} \\ \lambda_c^{<R_\alpha>} & \lambda_c^{<T_\alpha>} & 1 \end{pmatrix}. \quad (2.16)$$

The  $< . >$  symbol means summation over  $\alpha$ . To obtain the matrices  $\tilde{V}$  in the squark sector, all we have to do is to replace  $< . >$  by  $< |. | >$ , where  $|. |$  means absolute value. We can now compute for example

$$m_{12}^{R_d} = \min \left[ < (R_\alpha - T_\alpha) >, < |R_\alpha - T_\alpha| >, < (R_\alpha + |T_\alpha|) > \right] =$$

$$= < (R_\alpha - T_\alpha) > = < R_\alpha > - < T_\alpha > = 1 - 0 = 1. \quad (2.17)$$

By looking at 2.14, we see that the fnc constraints are not satisfied. In fact, in order to have a hope for satisfying the alignment constraints,  $Y_{d_{12}}$  has to be a holomorphic zero, so that in 2.17, the factor  $< (R_\alpha - T_\alpha) >$  that causes the misalignment is missing. To see this, notice that in order to reproduce the correct  $CKM$  matrix and intrafamily hierarchy, we have to have  $K_\alpha, L_\alpha > 0$  and  $N_\alpha^{[d]} > 0$  respectively. Similarly, from 2.13, we conclude that at the same time  $Y_{d_{21}}$  has also to be a zero. But if these are zeros, then also  $Y_{d_{32}}$  and  $Y_{d_{31}}$  have to be holomorphic zeros as well, as the sum rules 2.9 indicate. This is the minimum number of supersymmetric zeros in the down sector and it is also the maximum since neither the diagonal elements, nor  $Y_{d_{13}}$  and  $Y_{d_{23}}$  can be zeros if  $Y_d$  should give the desired mass ratios and mixings. We have therefore showed that there is a unique  $Y_d$  compatible with the “alignment scenario”

of suppressing fnc. On the other hand,  $Y_u$  is fixed except the elements (21), (31) and (32). These are either supersymmetric zeros or not and we thus verified the matrix forms in 1.3.

As we mentioned in the introduction, vacuum stability arguments suggest that we prefer models with the least number of holomorphic zeros and therefore we now turn to the analysis of those models.

### 3 No holomorphic zeros, dilaton stabilization and supersymmetry breaking

For a supersymmetry breaking mechanism with gaugino condensation in the hidden sector[6], in the presence of an anomalous  $U(1)$  [7] and where the dilaton dominates, it has been shown [8] that the dilaton can be stabilized with a weakly coupled Kähler potential, so that its second derivative  $K_2$  at the minimum of the scalar potential is very small. In this scheme, however, the  $D$ -term contribution to the soft masses is generically rather large [9]. The purpose of this section is to propose a mechanism which stabilizes the dilaton at a realistic value and at the same time suppresses the  $D$ -term relatively to the dilaton  $F$ -term. The mechanism that does both of the above mentioned things simultaneously, as far as we know, is new. To achieve this, we will assume a Kähler potential which is a combination of the one proposed in [8] and of a similar one to the one used in [9]. We will see that it will be necessary to have a  $K_2$  of order one in our scheme.

Let us first construct a model with no holomorphic zeros in the quark sector and a single  $U(1)$ . There is, in this case only one  $\theta$ -field ( $\theta_0 \equiv \theta$ ). We normalize the charge of  $\theta$  to be 1, which gives  $\mathcal{A} = 1$ . Then, noticing that the data implies  $K = 3, L = 2, M = 5, P = 2, R = 1$  and  $T = 0$ , equations 2.6 to 2.10, give

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4/3 \\ -1/3 \end{pmatrix}, \quad (3.18)$$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} -8/3 \\ 1/3 \end{pmatrix}, \quad (3.19)$$

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2/3 \\ 1/3 \end{pmatrix}. \quad (3.20)$$

We summarize in the following table the traceless part of our  $U(1)$ :

	<b>Q</b>	$\bar{\mathbf{u}}$	$\bar{\mathbf{d}}$
	$\begin{pmatrix} -4/3 \\ -1/3 \\ 5/3 \end{pmatrix}$	$\begin{pmatrix} -8/3 \\ 1/3 \\ 7/3 \end{pmatrix}$	$\begin{pmatrix} -2/3 \\ 1/3 \\ 1/3 \end{pmatrix}$

Therefore, the family dependent part of the symmetry acting upon the quark sector may be written as [1]

$$Y_F = B(2, -1, -1) - 2\eta(1, 0, -1), \quad (3.21)$$

where  $B$  is baryon number,  $\eta = 1$  for both  $\mathbf{Q}$  and  $\bar{\mathbf{u}}$  and  $\eta = 0$  for  $\bar{\mathbf{d}}$ . The next step is to supplement the quark sector by a lepton sector and/or a vector like sector such that the anomalies involving hypercharge that ought to vanish, vanish. To our knowledge, this is possible to do in a phenomenologically consistent way, only via an  $E_6$  embedding and specifically in the manner described in [1]. We will not repeat this analysis here. We only mention that the final result is the family  $U(1)$  called  $Y_X$  in the introduction. If we gauge  $X$ ,  $Y^{(1)}$  and  $Y^{(2)}$  separately, we have to supplement the visible sector of the model with a hidden sector as in [2], due to the appearance of the mixed anomaly  $XY^{(1)}Y^{(2)}$ , which is nonzero and has to be canceled. We will take this as a possible indication for the existence of a hidden sector. The ratio of the  $D$ -term mass squared over the dilaton  $F$ -term mass squared can be written in terms of the derivatives of the Kähler potential  $K_i$ ,  $i = 1, 2, 3$  as [8]

$$\frac{m_D^2}{m_{F_S}^2} = \frac{[-\frac{K_3}{K_1} + (\frac{K_2}{K_1})^2]}{2K_2(1 - 4\pi^2\delta_{GS}\frac{K_2}{K_1})}, \quad (3.22)$$

where  $\delta_{GS} = \text{Trace}(X)/(192\pi^2)$ . The task we have to carry out at this point is twofold.

The first is to propose a dilaton stabilization scenario with a specific Kähler potential, which stabilizes the dilaton at a value consistent with the value of the visible sector gauge couplings at the unification point (in [2], the latter was computed to be  $\sim 3/2$ ). This can be achieved



with a purely weak coupling  $K$  [8]:

$$K = -\log(2y) - \frac{2s_0}{(2y)} + \frac{(4s_0^2 + b)}{6(2y)^2}, \quad (3.23)$$

where  $y$  is the real part of the dilaton field  $S$  and  $s_0$  and  $b$  are numbers. Then, the assumption of dilaton dominance, amounts to the scalar potential of being simply

$$V = \frac{1}{K_2}|W_1|^2, \quad \text{with} \quad K_2 = \frac{1}{y^4}[(y - s_0)^2 + \frac{b}{4}]. \quad (3.24)$$

$W_1$  is the first derivative of the (strong coupling) superpotential  $W \sim e^{-mS}$  with respect to  $y$  ( $m$  is a numerical constant that can be computed in a specific model). Clearly, for  $b \rightarrow 0$ , if  $y \rightarrow s_0$  then  $K_2 \rightarrow 0$  and therefore  $V$  approaches infinity. Due to the exponentially decreasing form of  $W$  (if  $m > 0$ ), the dilaton will roll down the hill until it hits the bump located at  $s_0$ , provided  $b$  is small. In fact, the smaller  $b$  is, the higher and narrower the bump becomes. This will stabilize the dilaton at a value very close to  $s_0$ . Let us denote that value by  $y_0 \equiv s_0 - \alpha$ , with  $\alpha$  being a small positive number.

The second task is to explain why  $m_D^2 \ll m_{F_S}^2$ , which is equivalent to  $R_m \equiv -K_3/K_1 + (K_2/K_1)^2 \sim 0$ . Using (3.23), one can verify that  $K_2$  evaluated at the minimum of the potential is a very small number:  $K_2 = 1/(m^2 s_0^2)$ , which gives  $R_m \sim s_0^2/b$ , a rather large number. We see that a small  $K_2$  pushes  $R_m$  high, so a  $K_2$  of order 1 is desirable. We conclude that (3.23) is not sufficient for our model. To surmount this problem, we assume that the Kähler potential develops strong coupling contributions [10], [11], [9], just as the superpotential does. The origin of these strong coupling contributions can be the confining gauge group of the hidden sector or other strong coupling phenomena at higher energy. Here, we will assume a Kähler potential of the form:  $K^{\text{tot}}(y) = K(y) + K^{\text{np}}(y)$ , where

$$K^{\text{np}}(y) = ky^p e^{-ry^q}. \quad (3.25)$$

In the above,  $p$ ,  $q$  and  $r$  are unknown numbers ( $r, q > 0$ ). The constant  $k$  can be fixed from the equation  $(\xi/M)^2 = 4\pi^2 \delta_{GS} K_1^{\text{tot}}(y_0)$ , where  $\xi$  is the scale at which  $X$  breaks. Of course, the form of  $K^{\text{np}}(y)$  could be more complicated or simply different, but in any case, we will give arguments that indicate that it has to be some kind of exponential function. Having  $K^{\text{tot}}(y)$

as our starting point, we will try to answer the question, if it is possible to stabilize the dilaton in a similar fashion as with just  $K(y)$  and in addition to force  $R_m$  to zero. This will have to involve in our scheme a  $K_2^{\text{tot}}(y_0)$  of order of 1. We can express the above requirements with the following equations:

$$K_2^{\text{tot}}(s_0) \simeq 0, \quad (3.26)$$

$$K_2^{\text{tot}}(y_0) \simeq 1, \quad (3.27)$$

$$R_m \sim 0 : \quad K_1^{\text{tot}}(y_0)K_3^{\text{tot}}(y_0) \simeq K_2^{\text{tot}}(y_0)^2. \quad (3.28)$$

These are three constraints for three unknowns to be determined. Unfortunately, the equations are very nonlinear so we are not guaranteed a unique solution (not even a solution). In equations 3.26 and 3.27, we already see the seed of the reason for which we advocated that  $K^{\text{np}}(y)$  is some exponential function. In order for  $K_2^{\text{tot}}(y)$  to increase from its value 0 at  $y = s_0$  to 1 at  $y = s_0 - \alpha$  with  $\alpha$  very small, it most probably has to be an exponential function. We next give the derivatives of the Kähler potential:

$$K_1^{\text{tot}}(y) = -\frac{1}{y} + \frac{s_0}{y^2} - \frac{4s_0^2 + b}{12y^3} + \left(\frac{p}{y} - r q y^{q-1}\right) K^{\text{np}}(y), \quad (3.29)$$

$$K_2^{\text{tot}}(y) = \frac{1}{y^4} \left\{ (y - s_0)^2 + \frac{b}{4} + (r q y^q)^2 \left[ \left( y - \left( \frac{p}{r q y^{q-1}} - \sqrt{\frac{p}{(r q y^{q-1})^2} + \frac{(q-1)}{r q y^{q-2}}} \right) \right. \right. \right. \\ \left. \left. \left( y - \left( \frac{p}{r q y^{q-1}} + \sqrt{\frac{p}{(r q y^{q-1})^2} + \frac{(q-1)}{r q y^{q-2}}} \right) \right) K^{\text{np}}(y) \right] \right\}, \quad (3.30)$$

$$K_3^{\text{tot}}(y) = -\frac{2}{y^3} + \frac{6s_0}{y^4} - \frac{4s_0^2 + b}{y^5} + \left(\frac{p}{y} - r q y^{q-1}\right) K_2^{\text{np}}(y) - \\ 2\left(\frac{p}{y^2} + r q (q-1) y^{q-2}\right) K_1^{\text{np}}(y) + \left(\frac{2p}{y^2} - r q (q-1)(q-2) y^{q-2}\right) K^{\text{np}}(y). \quad (3.31)$$

For the constant  $k$ , we obtain:

$$k = \frac{\frac{\lambda_c^2}{4\pi^2 \delta_{GS}} + \frac{1}{y_0^3} \left( y_0^2 - s_0 y_0 + \frac{4s_0^2 + b}{12} \right)}{y_0^p \left( \frac{p}{y_0} - r q y_0^{q-1} \right) e^{-r y_0^q}}, \quad (3.32)$$

where we have set  $\xi/M \sim \lambda_c$ . Defining  $x_y \equiv r q y_0^q$  and  $x_s \equiv r q s_0^q \simeq x_y$ , after a considerable amount of algebra and always keeping only the dominant contributions, we find that provided that  $\alpha$  is a small number, equations 3.26-3.28 reduce to

$$x_s = p + \frac{1}{2}(q-1) \pm \frac{1}{2} \sqrt{(q-1)^2 + 4qp}, \quad (3.33)$$

$$x_y \frac{2\hat{\lambda}\alpha}{y_0^2} = 1, \quad (3.34)$$

$$\frac{\lambda_c^2}{4\pi^2\delta_{GS}} \frac{(-2q\hat{\lambda})}{y_0^2} x_y = 1 \quad (3.35)$$

respectively, where  $\hat{\lambda} = \frac{\lambda_c^2}{4\pi^2\delta_{GS}} + \frac{b}{12y_0^3} + \frac{1}{3y_0} - \frac{\alpha}{3y_0^2} + \frac{\alpha^2}{3y_0^3} \simeq \lambda_c$ . If we recall [2] that  $y_0$ ,  $\delta_{GS}$  and  $\lambda_c$  are calculable numbers, we see that the parameters  $p, q$  and  $r$  can be derived from 3.33 to 3.35, which shows that there exist  $p, q$  and  $r$  that satisfy 3.26-3.28. However, since we do not have any physical intuition at the moment what are the expected values of these parameters, we will not give specific examples. The lesson from this analysis is that we need to have  $K_2^{tot}(y_0) = 1$  to suppress fnc and that this is possible only if the Kähler potential has some additional contributions, probably due to strong coupling physics. In a future work we will investigate the different possibilities and constraint them by looking at cosmological issues<sup>3</sup>. Fortunately, we do not need to have an explicit form for  $K^{tot}$  in order to make predictions about low energy physics, as long as we require that whatever form it has, it is such that  $K_2^{tot}(y_0) = 1$ .

The superpotential for a QCD like hidden sector (that develops gaugino condensates) with gauge group  $SU(N_c)$  and  $N_f$  families ( $N_f < N_c$ ) is

$$W = \frac{1}{2} M t^2 \left( \frac{\theta_0}{M} \right)^{p_0} \left( \frac{\theta_1}{M} \right)^{p_1} \left( \frac{\theta_2}{M} \right)^{p_2} + (N_c - N_f) \left( \frac{2\Lambda^{\frac{\beta_0}{2}}}{t^2} \right)^{\frac{1}{N_c - N_f}}, \quad (3.36)$$

where  $t$  is the “quark” condensate and  $\beta_0 = 2(3N_c - N_f)$  is the one loop  $\beta$  function. Also,  $\Lambda = M e^{-8\pi^2 k_h (2S)/\beta_0}$ , with  $k_h$  the Kac-Moody level of the hidden group and [7],

$$\hat{m} = M \left( \frac{\langle \theta_0 \rangle}{M} \right)^{p_0} \left( \frac{\langle \theta_1 \rangle}{M} \right)^{p_1} \left( \frac{\langle \theta_2 \rangle}{M} \right)^{p_2} \quad \text{and} \quad \epsilon = \left( \frac{\Lambda}{\xi} \right)^{\frac{\beta_0}{2N_c}} \left( \frac{\xi}{\hat{m}} \right)^{1 - \frac{N_f}{N_c}}. \quad (3.37)$$

The  $F$ -term contribution to the soft masses then is [8], [12]

$$m_0^2 = \left[ \frac{2\sqrt{2}}{\sqrt{K_2^{tot}(y_0)}} (\epsilon \hat{m}) (8\pi^2 k_h) \lambda_c^2 \right]^2, \quad m_{1/2} \simeq m_0 \simeq a_0 \quad (3.38)$$

and the Kähler potential dependence only comes through  $K_2^{tot}$  at the minimum, which as we argued, is equal to 1.

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<sup>3</sup>An alternative, interesting form that will be examined from this point of view in a future work will be  $K^{np}(y) = c_1 + c_2 \int_0^y e^{-4\pi^2(t-s_0)^2} dt$ , which along with  $K^0(y) = -\ln(2y) - \frac{(s_{-1}+s_0)}{2y} + \frac{2/3s_{-1}s_0}{(2y)^2}$  ( $s_{-1}$ ,  $c_1$  and  $c_2$  are constants), stabilizes the dilaton at  $s_0$ .

Table 1: Higgs mass versus  $\tan\beta$  for different values of  $M$ . The second column is for  $M \simeq M_{GUT} = 4 \cdot 10^{16}$  GeV, the third for  $M = 8 \cdot 10^{16}$  GeV and the fourth for  $M = 1.2 \cdot 10^{17}$  GeV.

$\tan\beta$	$h_0$ (GeV)	$h_0$ (GeV)	$h_0$ (GeV)
2	79.0	90.0	95.0
3	80.2	92.0	97.0
4	91.5	103.0	107.0
5	99.6	109.5	112.0

We are now ready for an explicit numerical example. We first recall the values of the hidden sector parameters (which are the relevant ones for our case too) found in [12]:  $N_c = 5$ ,  $N_f = 3$ ,  $\delta_{GS} = -0.113$ ,  $p_0 = p_1 = p_2 = 6$ ,  $k_h = 1$ ,  $\langle \theta_i \rangle / M = 0.222 \simeq \lambda_c$ . Second, we recall that in the same model, the scale at which the gauge couplings unify is  $M_{GUT} \simeq 4 \times 10^{16}$  GeV and the vacuum expectation value of the dilaton, (the one loop unified gauge coupling at  $M_{GUT}$ ) is  $y_0 = 1/g^2(M_{GUT}) = 1.429$ , Third, we will choose the sign of the  $\mu$  term to be positive, since the negative choice tends to give problems with vacuum stability. Given the above, we end up having only two free parameters:  $\tan\beta$  and the cut off scale  $M$ , which need not be necessarily be  $M_{GUT}$ . In addition, if we remind ourselves that in this model we have  $m_b/m_t \sim \lambda_c^3$ , we conclude that  $\tan\beta$  can not be larger than about 5 if we do not want to introduce unnaturally small numerical coefficients in front of the Yukawa couplings. Having  $m_0$ ,  $m_{1/2}$ ,  $a_0$  (from 3.38),  $\tan\beta$  and  $sgn(\mu)$ , we can make specific low energy predictions if we run the parameters via the RGE equations. We present a table with the prediction of the Higgs mass for different values of  $\tan\beta$  and  $M$ .

We close this section by giving arguments to show the fact that supergravity effects will have negligible corrections to this picture. The ratio  $R_m$  including supergravity corrections can be written as [9]

$$R_m^{\text{SUGRA}} = \frac{\left[ -\frac{K_3^{tot}}{K_1^{tot}} + \left(\frac{K_2^{tot}}{K_1^{tot}}\right)^2 + \Delta_2 \left(\frac{K_2^{tot}}{K_1^{tot}}\right)^2 - 4\pi^2 \delta_{GS} (\Delta_1 + 1) \left(\frac{K_2^{tot}}{K_1^{tot}}\right)^3 \right]}{2K_2^{tot} \left(1 - 4\pi^2 \delta_{GS} \frac{K_2^{tot}}{K_1^{tot}}\right)} + \frac{\Delta_3}{K_2^{tot}} \left(\frac{K_2^{tot}}{K_1^{tot}}\right)^2, \quad (3.39)$$

where (for  $N_f = 1$ ):

$$\Delta_1 = \frac{z^2 + 6nz - 4n^2}{(z - 2n)^2}, \quad \Delta_2 = \frac{2z^2 + 2nz}{(z - 2n)^2}, \quad (3.40)$$

where  $z = 2N_c\lambda_c^2$  and  $\Delta_3$  is given by [9]

$$\Delta_3 = \frac{1}{2}K_1^{tot}(\tilde{\delta}_{GS} - 4\frac{K_1^{tot}}{K_2^{tot}}) - \frac{N_c}{n^2}\frac{\tilde{\delta}_{GS}K_1^{tot^2}}{4K_2^{tot}}(-nK_1^{tot} + \frac{1}{4}(N_c + n)\tilde{\delta}_{GS}K_2^{tot}) - \frac{N_c}{n}\frac{3\tilde{\delta}_{GS}K_1^{tot}}{4}, \quad (3.41)$$

where  $\tilde{\delta}_{GS} = 16\pi^2\delta_{GS}$ . We can easily check that  $\Delta_1 \simeq -1$ ,  $\Delta_2 \simeq 0$  and  $\Delta_3 \simeq 0$  which demonstrates that the global supersymmetry limit was sufficiently good for our numerical predictions, since only equation 3.28 is modified slightly from  $R_m \sim 0$ , to  $R_m^{\text{SUGRA}} \sim 0$ .

## 4 Conclusions

We have shown that in models with  $U(1)$  family symmetries the alignment mechanism can be implemented via the appearance of holomorphic zeros in the superpotential -which tend to destabilize the vacuum by opening flat directions. On the alternative, there can be constructed models of fermion masses with minimal number of holomorphic zeros which in order to be viable from the fnc point of view are complemented by a hidden sector and a dilaton dominated supersymmetry breaking mechanism. Such a mechanism that at the same time stabilizes the dilaton and suppresses the  $D$ -term contributions to the soft masses is presented. An explicit numerical example demonstrated that the model gives a small Higgs mass, very close in some cases to its current central value. Even though it is true that a low Higgs mass is a generic feature of supersymmetric models, we think that it is remarkable that a model that has such a few free parameters, gives predictions in the expected range.

## 5 Acknowledgements

I would like to thank P. Ramond for discussions and his useful comments and R. Haas for his help with SPYTHIA. NI is supported in part by the United States Department of Energy under grant DE-FG02-97ER41029.

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